

Persistent Non-ergodic fluctuations in mesoscopic insulators

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Abstract

We give a detailed and rigorous picture of the mesoscopic conductance fluctuations in the deep insulating regime (DIR) within the Nguyen, Spivak and Shklovskii model including spin-orbit coupling (SO). Without SO, we find that fluctuations of the log-conductance are persistent above a saturation field B_s , where one has that the log-conductance is approximately a stationary random process. In contrast, in the SO case the saturation field B_s is negligible and the stationarity is well realized. We find non-vanishing disorder fluctuations of the field average of the log-conductance as a quantitative measure of the lack of ergodicity in the mean square sense. To this fact, a weak decaying behavior of the correlation function, even weaker in the case of SO is established on the relevant field scale of the model, one flux quantum per plaquette. This finding corroborate the behavior of the fluctuations of the field average of the log-conductance and permit us to invoke Slutski's theorem to conclude that the whole stochastic process defined by the log-conductance is non-ergodic in the mean square

sense in both cases. As a consequence the commonly used criterion to test the ergodicity based on the equivalence of the variance in disorder and the variance in the field is not fulfilled. Using the replica approach, we derive the weak localization analogs of the ‘cooperon and diffuson which permits us to analyze in qualitative form the decaying behavior of the correlation function. Our predictions agree qualitatively and semi-quantitative with experiments in the DIR.

1 Introduction

The nature of Fluctuations in both the metallic state [1],[2], and in disordered insulators [3], [4, 5], has been a matter of interest for both theoretical and experimental studies. Whereas in the metallic regime the basic aspects of fluctuations have been elucidated, in the regime of hopping transport the nature of fluctuations is still an open field. The deep insulating regime, DIR, where transport occurs via variable range hopping (VRH), is defined as the regime where the localization length is the smallest scale compared to the elastic mean free path and hopping lengths, i.e., $\xi < \ell < t$ respectively[6]. Coherence effects are possible in this regime because phase breaking events occur at the hopping length[7], which is larger than ℓ . Important signatures of quantum interference in disordered insulators are the classic magneto-fingerprints, or reproducible fluctuations in the conductance with magnetic field, and a low field positive magneto-conductance.

An important property of mesoscopic conductance fluctuations in the metallic phase is their ergodicity. At the mesoscopic level, the sample size is less than thermal diffusion length or the dephasing length, whatever is shorter, such that sample to sample fluctuations are visible and the system does not self-average. Although it was not rigorously proven, the ergodic hypothesis was meant as the ability of the magnetic field (or energy) to induce conductance fluctuations equivalent to sample to sample fluctuations (Lee-Stone criterion) [1]. In contrast, experimental results show that log-conductance mesoscopic fluctuations in the DIR without spin-orbit scattering are not ergodic in the Lee-Stone sense[4, 5, 8], i.e., the variance over samples is larger than the variance over field. Such samples involve hopping lengths that are, at most, 6 to 10 times the localization length. Precise measurements of Ladiou et al[4] and Orlov et al[5] have shown that a) field fluctuations do not decorrelate disorder fluctuations, b) field fluctuations do not change the identity of the hop, c) the field average of the variance over the samples is larger than the sample average of the variance over the field and d) there exist a decorrelation field B_c defined by the field correlation function, which defines a equivalent new sample.

The question of the ergodic nature of fluctuations with and without SO has not been addressed, to our knowledge, from the theoretical side. Our plan of this work is first to address the problem of fluctuations and the question of ergodicity in DIR within the NSS model. We undertake this task through the verification of concepts concerning ergodicity, which we first define with

mathematical rigor in the next mostly technical section. Then in section 3 we explain the NSS model and define the random processes we want to analyze. In section 4 we carry out the program described in section 2 and verify the non-ergodic behavior of fluctuations. In section 5 we define the main theoretical objects of this work, the cooperon and diffuson analogs of weak localization theory, with the help of which, we can explain to some degree the decaying behavior of the correlation function. Finally we conclude by discussing and comparing our results with experiments.

2 Ergodicity of Transport Fluctuations

In the preceding section one we introduced the question of ergodicity of fluctuations. Here we introduce the mathematical concepts that will permit us to establish the ergodic nature of the fluctuations.

Given a physical quantity $F(\mathcal{H}, B)$ depending on the disordered Hamiltonian \mathcal{H} and magnetic field B , we denote by $\overline{F(\mathcal{H}, B)}$ the sample to sample average, or disorder average, and by $\langle F(\mathcal{H}, B) \rangle = \Delta B^{-1} \int_{B_i}^{B_f} dB F(\mathcal{H}, B)$ the field average for a given sample or disorder realization. In order to be able to estimate the sample average from the field average of a given sample the following conditions must be satisfied: a) $\lim_{B_f \rightarrow \infty} \sigma_{mss}(B_f) = \overline{[F(\mathcal{H}, B) - \langle F(\mathcal{H}, B) \rangle]^2} \rightarrow 0$ and b) $\overline{F(\mathcal{H}, B)} = \langle \overline{F(\mathcal{H}, B)} \rangle$. The verification of both conditions is known as *ergodicity in the mean square sense* (mss), or the random function $F(\mathcal{H}, B)$ is said to be ergodic in the mean-square limit[9]. The condition $\overline{F(\mathcal{H}, B)} = \langle \overline{F(\mathcal{H}, B)} \rangle$ is a measure of global stationarity. which means that these averages are independent of B . One can cast conditions a) and b) into a single statement on the disorder fluctuations of the field average, i.e., $\lim_{B_f \rightarrow \infty} \sigma_{mss}(B_f) = \lim_{B_f \rightarrow \infty} \text{Var}_d(\langle F(\mathcal{H}, B) \rangle) = \overline{(\langle F(\mathcal{H}, B) \rangle - \overline{\langle F(\mathcal{H}, B) \rangle})^2} \rightarrow 0$, where Var_d means variance over disorder[10]. This property implies that for one realization of disorder there are enough equivalent samples within the magnetic scale, such that the average in the field, with regard to disorder, does not depend in statistical sense on the particular realization. This means that $\overline{\langle F(\mathcal{H}, B) \rangle} \approx N \langle F(\mathcal{H}, B) \rangle$, where N is the number of realizations, and a sharp distribution of $\langle F(\mathcal{H}, B) \rangle$ over disorder holds.

One could also ask for the possibility of making estimates of $\text{Var}_d(F(\mathcal{H}, B))$ from $\text{Var}_B(F(\mathcal{H}, B))$, (here $\text{Var}_B(F(\mathcal{H}, B))$ means the variance over the

field B of $F(\mathcal{H}, B)$), or more generally, try to make an estimate of another function of the basic process $F(\mathcal{H}, B)$, $g(F(\mathcal{H}, B))$. The necessary and sufficient conditions such that one can estimate $\overline{g(F(\mathcal{H}, B))}$ from one realization of disorder with the field average $\langle g(F(\mathcal{H}, B)) \rangle$ (the so called law of strong numbers), is given by Slutski's theorem. One can write: $\sigma_{mss}(B_f) = \lim_{B_f \rightarrow \infty} \frac{2}{\Delta B^2} \int_{B_i}^{B_f} dB (B_f - B) C(g(F(\mathcal{H}, B)))$, with:

$$C(g(F(\mathcal{H}, B, \Delta B))) = \overline{\Delta[g(F(\mathcal{H}, B + \Delta B)) \times \Delta[g(F(\mathcal{H}, B))]} \quad (1)$$

where $C(g(F(\mathcal{H}, B, \Delta B)))$ is the correlation function for $g(F(\mathcal{H}, B, \Delta B))$ and $\Delta[g(F(\mathcal{H}, B + \Delta B))] = [g(F(\mathcal{H}, B + \Delta B)) - \overline{g(F(\mathcal{H}, B + \Delta B))}]$. One can easily realize that a strong decaying behavior of the correlation function with correlation length $\tau \ll B_f$ will be a sufficient condition for ergodicity in the mss, i.e. $\sigma_{mss}(B_f) \Rightarrow 0$. In fact, it is also a necessary condition.

Usually, for the application of the theorem stationarity in the wide sense of $F(\mathcal{H}, B)$ can be assumed, i.e., $\overline{F(\mathcal{H}, B)}$ does not depend on B and $C(g(F(\mathcal{H}, B, \Delta B)))$ should depend only on ΔB [11]. In this work we are interested in testing the ergodicity in two cases: $g(X) = X$ for the average and $g(X) = X^2 - \overline{X}^2$ for the variance. The first case corresponds to the usual meaning of ergodicity in statistical mechanics. The second case, usually named the Lee-Stone criterion, refers to the equivalence of sample to sample fluctuations and magnetic field fluctuations. [1][9][12].

3 The NSS Model

We examine now the fluctuations in the DIR in two and three dimensions and define the random process $F(\mathcal{H}, B)$. This is obtained from the Nguyen, Spivak and Shklovskii[6, 7] model (NSS). The NSS model's crucial insight is that coherence is maintained within a Mott hopping length, where the conductance is a sum of coherent *forward directed Feynman paths* which interfere with each other. The NSS model describes the quantum behavior of the critical (bottleneck) hop in the Miller-Abrahams network[13]. The existence of many randomly oriented critical hops tend to average the macroscopic conductance, eliminating fluctuations[16]. Here, we focus on the low temperature regime where critical hops do not trivially self average[8][17], i.e., the percolation correlation length ξ_p is such that $\xi_p = \xi(T_o/T)^{(\nu+1)/(D+1)} \sim L$, where ν is the percolation correlation length exponent, D is the spatial dimension and T_o a disorder parameter. This is the mesoscopic regime [8]. We

will first find the fields above which the process $F(\mathcal{H}, B)$ can be considered stationary and test the ergodicity criterion in the mean square limit. We then calculate the correlations functions and their decaying behavior. Invoking Slutski's theorem, we confirm non-ergodic behavior as established by the finiteness of the mean square criterion, $(\lim_{B \rightarrow \infty} \text{Var}_d(\langle F(\mathcal{H}, B) \rangle)) \neq 0$. We then proceed to test the ergodicity of fluctuations. Non-ergodicity of fluctuations is implied by the weakly decaying behavior of the correlation function. We find this consistently. Here, we find good quantitative agreement with the measurements of Orlov et al [5]. We derive the cooperon and diffuson analogs of the weak localization theory for DIR with the help of which the non ergodic behavior can be explained. Furthermore, the predictions for the case with SO are made.

In the two dimensional NSS model, impurities are placed on the sites of a lattice of main diagonal length t (the hopping length, $t = \xi(T_o/T)^{1/(D+1)}$, Mott's law). We apply a magnetic field B , perpendicular to the plane, changing only the phases of the electron paths. The overall tunneling amplitude is computed by summing all forward directed paths between two diagonally opposed points, each contributing an appropriate quantum mechanical complex 2×2 matrix weight given by the Hamiltonian:

$$\mathcal{H} = \sum_i \epsilon_i a_{i,\sigma}^\dagger a_{i,\sigma} + \sum_{\langle ij \rangle_{\sigma,\sigma'}} V_{ij,\sigma,\sigma'} a_{i,\sigma}^\dagger a_{j,\sigma'}, \quad (2)$$

where ϵ_i is the site energy, and $V_{ij,\sigma,\sigma'}$ represents the nearest neighbor couplings or transfer terms which includes a randomly chosen SU(2) matrix describing a spin rotation due to strong SO scattering. Within the NSS model, we choose site energies to be $\epsilon_i = \pm W$ with equal probability [7] [24]. Without SO the coupling terms are diagonal in spin space $V_{ij} = V$, and the Green's function between the initial and final site is given by

$$\langle i | G(E) | f \rangle = \left(\frac{V}{W} \right)^t J(B, t); \quad J(B, t) = \sum_{\Gamma'}^{\text{directed}} \left[\prod_{i_{\Gamma'}} \eta_{i_{\Gamma'}} e^{i\phi_{i_{\Gamma'}}} \right], \quad (3)$$

where $\phi_{i_{\Gamma'}}$ is the phase gained through path $i_{\Gamma'}$ due to the magnetic vector potential, Γ' represents all directed paths that go from i to f through the lattice and $\eta_i = \text{sign}(\epsilon_i) = \pm 1$ [25]. In the presence of spinorbit scattering the Green's function is the 2×2 matrix:

$$J(B, t) = \sum_{\Gamma'}^{\text{directed}} \left[\prod_{i_{\Gamma'}} \eta_{i_{\Gamma'}} \right] \left[\prod_{i_{\Gamma'}} U_{i_{\Gamma'}} \right] e^{\sum i\phi_{i_{\Gamma'}}}, \quad (4)$$

The Green's function consist of sum of terms, one for each path, each being a product of random numbers and random SU(2) matrices, and a deterministic disorder independent phase factor from the magnetic vector potential[23]. The complex function $J^2(B, t)$ (a complex matrix function in the presence of SO) contains the interference information including correlations due to crossing of paths, and the factor $(V/W)^t$ is the leading contribution to the exponential decay of the localized wavefunction. We use the transfer matrix approach in order to compute $J(B, t)$, exactly, for each realization of disorder[24]. Our random processes in question represent the log-conductance. They are $F(\mathcal{H}, B) = \ln(J^\dagger(B, t)J(B, t))$ and with SO $F(\mathcal{H}, B) = \ln(I(B, t))$, where $I(B, t) = 1/2\text{Tr}(J^\dagger(B, t)J(B, t))$ [26]. In his work we measure the magnetic field B or changes in magnetic field ΔB in flux units ϕ_o/ℓ^2 , where ϕ_o is the flux quantum.

4 Fluctuations and Ergodicity

Figure 1 shows typical fluctuations of the log-conductance as a function of the sample and the magnetic field. Without SO, the figure clearly shows that the average of $\ln(J(B, t))$ dominates the fluctuations, i.e., the average behavior is visible for a single sample. As the average $\ln(J(B, t))$ first increases proportional to B , crossing over to a slower growth as $B^{1/2}$ dictated by the magnetic length $\Delta B < B_c = \pi c\hbar/(\xi^{1/2}e t^{3/2})$ [24, 32, 27], the process is not stationary. However, in the latter regime of slow growth, one finds a field above which the process can be considered as essentially stationary in the same fashion as in the metallic regime. In this regime, while the log-conductance tends to saturate, the fluctuations persist as in mesoscopic fluctuation theory in metals[1]. Furthermore, we note that the average behavior is periodic in half the flux quantum ϕ_o per ℓ^2 (only one half of a period is shown). This periodicity reveals an average field coupling to $2B$ [7] which has been demonstrated theoretically[31]. In three dimensions, the fluctuations are appreciably larger than the average behavior. Once more, persistent fluctuations beyond the average conductance saturation field are observed. The existence of such persistent fluctuations were first surmised by Sivan et al[16, 28] and Zhao et al[27]. With SO, there is no tendency to build an average, and there are in general soft changes in the fluctuations in marked contrast with the sharp changes without SO. This peculiarity will be explained later. One marked feature of figure 1(with and without SO), are

the fact that disorder fluctuations do not decorrelate the field fluctuations, which is the reason of the remarkable similarity with fig.2 of the work of [5]. The key point in both figures is that the the fluctuations do no decorrelate at the scale of fields shown in the figures, suggesting non-ergodic behavior.

Following the concepts of section two, we analyze the ergodicity of the log-conductance fluctuations more carefully. To achieve this we have to check two points: first if stationarity is reasonably fulfilled and second, we have to verify the ergodicity condition and the decaying behavior of the correlation function. Fig.2a shows the quantities $\text{Var}_d(\langle F(\mathcal{H}, B) \rangle)$ (stars) and $[\overline{F(\mathcal{H}, B)} - \langle F(\mathcal{H}, B) \rangle]^2$ (diamonds) in the absence of SO ($F(\mathcal{H}, B) = \ln |J(B, t)|^2$). The field averaging interval is $\Delta B = [B_i, B_f]$. In this figure as the field B_i is increased from zero to 0.09 in $[\phi_0/\ell^2]$ units, the diamonds move downward overlapping the stars sooner for smaller t . Therefore, the field above which the process can be considered quasi stationary increases with the hopping length t such that both quantities tend to coincide for larger B_i . This feature is of importance for calculating other quantities, as any question on ergodicity presumes at least quasi-stationarity. In the case of SO ($F(\mathcal{H}, B) = \ln |I(B, t)|^2$), Fig.2b shows effective stationarity of the process $\ln(I(B, t))$, essentially independent of B_i and t . $\text{Var}_d(\langle F(\mathcal{H}, B) \rangle)$ does not tend to zero with increasing B_f ; on the contrary, it increases with a power of t [31], which is an indicator of non-ergodic behavior as there is no self-averaging as one increases the hopping length t (see Kramer and Mackinnon [2]). The last result establishes non-ergodic behavior in the mean square sense with and without SO. To further substantiate this result we calculate the correlation functions. Fig. 3a shows the correlation function for three values of the hopping length t and some values for B_i . One sees a very weakly decaying behavior on the physical field scale(ϕ_o/ℓ^2) and a tendency to decay faster for bigger B_i , indicating that when the process becomes quasi stationary there is a tendency to a faster decaying correlation function. Fig. 3b with SO shows that the correlations function depends essentially only on ΔB , and the decaying is even weaker with a functional form depending on t . Now, the basic argument against ergodicity is that the decaying behavior of the correlation functions is such that it is not possible to construct enough ensembles from the field fluctuations data and therefore non-ergodic behavior is established. To illustrate this point, we write $\sigma_{mss}(B_f) = \frac{2}{\Delta B^2} \int_{B_i}^{B_f} dB [(B_f - B)C(F(\mathcal{H}, B))]$. In order to have enough ensembles in the field scale within the validity of the model ($B_f \leq \phi_o/\ell^2 = 1$

in our units) , there should be a decorrelation field $B_c \ll 1$, such that large number of samples could be defined, so that one would get $\sigma_{mss}(B_f) \approx 0$ - could be satisfied. This, on the other hand, implies that the condition for ergodicity of the variance, which in general requires a stronger decaying behavior of the correlation function for process $F(\mathcal{H}, B)$ than the condition required for ergodicity in the mean square sense, is not at all fulfilled[9]. Therefore, one should expect that the Lee and Stone criterion of ergodicity on the relative magnitude of the field and sample fluctuations is not realized. To check this point one has to compare the magnitude of the variance in field and sample to sample fluctuations. The idea of further averaging over disorder and over the field, respectively, is the same as in statistical mechanics using different initial conditions to improve the statistics. Figure 4 shows the averages

$$\langle \text{Var}_d(\ln |J(B, t)|) \rangle = \overline{\langle (\ln |J(B, t)| - \overline{\ln |J(B, t)|})^2 \rangle}, \quad (5)$$

$$\overline{\text{Var}_B(\ln |J(B, t)|)} = \overline{\langle (\ln |J(B, t)| - \langle \ln |J(B, t)| \rangle)^2 \rangle}. \quad (6)$$

Without SO we find the characteristic B_i dependence shown in fig.2 shows up as a crossing of both types of averages. There is again a clear tendency to saturate as B_i increases whereas the difference of the saturated averages widens with increasing t . This tendency is clearly seen in the case with SO where there is a much weaker dependence on B_i as expected from fig.2 and the crossing observed in the previous case is absent.

5 The Correlation function: the Cooperon and Diffuson

In this section we develop the concepts of the cooperon and diffuson in the context of strong localization. These objects are used to explain qualitatively and semi-quantitatively non-ergodic behavior in the mean square sense and the relative magnitude of the field and sample fluctuations found in experiments. For this purpose, the following relation is straightforwardly derived :

$$\text{Var}_d[F(B+\Delta B, t)+F(B, t)] = \text{Var}_d[F(B+\Delta B, t)]+\text{Var}_d[F(B, t)]+2C(F(\mathcal{H}, B, \Delta B, t)) \quad (7)$$

This relation is valid for both processes $\ln J(B, t)$ and $\ln I(B)$ [14]. For the sake of clarity, the composite process inside the brackets on the left side of eqn.7 is denoted by the process $P(B, \Delta B, t) = [\ln |J(B + \Delta B)|^2 + \ln |J(B)|^2]$ and with SO by $P_{spinor}(B, \Delta B, t) = [\ln I(B + \Delta B) + \ln I(B)]$, such that the left hand side of eqn. 7 is in each case $\text{Var}_d[P(B, \Delta B, t)]$ and $\text{Var}_d[P_{spinor}(B, \Delta B, t)]$, respectively. These two functions are shown in fig. 5 as a function of ΔB in ϕ_0/ℓ^2 units and three values of t (30, 100, 300) from the bottom to the top, respectively). For B_i shown in the figure and the scales of ΔB used, three things deserve explanations: first for large enough ΔB , the above mentioned functions show almost no dependence on B_i and ΔB at fixed t ; secondly, the ratio $\text{Var}_d[P(B, \Delta B, t)] / \text{Var}_d[P_{spinor}(B, \Delta B, t)]$ is around two, for big enough t and ΔB which is a landmark of the symmetry changing from unitary to symplectic; Thirdly, one observes a very rapid decaying on the dependence on B , which can be traced to the saturation behavior of the cooperon as we will see.

Recall that on the metallic side, an analogous behavior has been described which identifies two fundamental contributions to the field effect: the cooperon and the diffuson[1][2]. These contributions can be distinguished by the way they enclose the magnetic flux; while the cooperon is sensitive to $(2B + \Delta B)$, the diffuson only responds to field changes ΔB . In the insulating regime, a mechanism similar to the cooperon which saturates is associated with a positive magneto-conductance (MC). This has been observed as a general effect[2][8, 15, 5, 18]. A semi quantitative explanation for the behavior of the functions $\text{Var}_d[P(B, \Delta B, t)]$ and $\text{Var}_d[P_{spinor}(B, \Delta B, t)]$ and their ratio can be found with the help of the cooperon and diffuson analogs. To achieve this goal, we consider the moments of process of $P(B, \Delta B, t)$ and $P_{spinor}(B, \Delta B, t)$ (further below it will become clear why). In the former case they are given by $[J^*(B + \Delta B)J(B + \Delta B)J^*(B)J(B)]^n$. Recall from equations 3 and 4 that this product can be visualized as a set of n paths, each one defined by the respective term in the product. These paths eventually intersect each other at some values of their length. In order to have nonzero contributions after disorder average, the paths must pair up, as a consequence of the chosen distribution of the energies [29]. Neutral paths (field independent) are formed by pairing J^* and J at the same field (phase cancels). On the other hand, charged paths (field sensible) are formed by pairing either $J^*(B + \Delta B)$ and $J(B)$ or $J^*(B + \Delta B)$ and $J^*(B)$. In the absence of paired path intersections, self interference kills charged paths (their contribution decays exponentially fast). Nevertheless, if intersections are con-

sidered, one can have path exchanges for short distances, yielding a magnetic field coupling which is analogous to the magneto-conductance, the source of the initial decaying of the correlation function. There are three possible diagrams at a paired path crossing, two of which are depicted in fig.7. Without SO the spin indexes can be ignored, one obtains [31]: a) one partner from $J^*(B + \Delta B)$ pairs with one from $J^*(B)$ while one from $J(B + \Delta B)$ and one from $J(B)$ follow a different path. Such a combination encloses $(2B + \Delta B)$ and is therefore called *cooperon*-like. b) One partner is taken from $J^*(B + \Delta B)$ and the other from $J(B)$ on the same path, while one from $J(B + \Delta B)$ and $J^*(B)$ follow another. Such a combination encloses only ΔB and is called *diffuson*-like. Finally, one can have combination c) where one partner comes from $J^*(B + \Delta B)$ and the other from $J(B + \Delta B)$, leaving $J^*(B)$ and $J(B)$ to pair up. The latter combination is called uncharged and encloses no field. Note that all previous cases satisfy overall neutrality so that the contributions are real as expected. The contribution of the replica cooperon and diffuson are the same at zero field and there is an additional contribution from the uncharged diagram. Further progress is achieved using the replica argument.

The replica-moment argument [29], maps the n -th moment problem onto the problem of $2n$ bosons with contact interaction. These interactions renormalize due to the diagrams above, making path interactions field dependent. The $2n$ boson system can be solved using the Bethe ansatz and has ground state energy $\epsilon_0 = \ln 4^n + \rho(B, \Delta B)n(n^2 - 1)$, where $\rho(B, \Delta B)$ is a function of $B, \Delta B$, such that one has $\overline{[J^*(B + \Delta B)J(B + \Delta B)J^*(B)J(B)]^n} = A(n, B, \Delta B) \exp(\ln 4^n + \rho(B, \Delta B)n(n^2 - 1)t)$ valid at fixed n asymptotically for $t \rightarrow \infty$. On the other hand, the n -th moment can be expressed as a cumulant expansion valid at fixed t asymptotically for $n \rightarrow 0$, $\overline{[J^*(B + \Delta B)J(B + \Delta B)J^*(B)J(B)]^n} = \exp\{\sum \frac{n^i}{i!} C_i[P(B, \Delta B, t)]\}$, where $C_i[P(B, \Delta B, t)]$ are the cumulants of process P . The subtleties concerning both limits have been discussed by Kardar [29], who finds a nonextensive correction subleading term proportional to $t^{2/3}$. We therefore obtain :

$$\text{Var}_d[P(B, \Delta B, t)] = (\rho_{coop}(2B + \Delta B) + \rho_{diff}(\Delta B))t^{2/3} + \ln A(B, \Delta B) \quad (8)$$

Here $\rho(B, \Delta B) = \rho_{coop}(2B + \Delta B) + \rho_{diff}(\Delta B)$, we have separated the path interaction in terms of the cooperon and diffuson contributions. We check numerically this important prediction and fig.6 shows the expected scaling

with an exponent near $\frac{2}{3}$. Beyond the saturation field of the average log-conductance, the cooperon term on the right hand side saturates (the same for the variance on the right of eqn. 7 which depend on B) and the correlation function only depends on ΔB . This behavior is summarized in Fig5.

In the case of SO, it can be shown[24], that the only non-zero paired averages are $\overline{U_{\alpha\beta}U_{\alpha\beta}^*} = \frac{1}{2}$, $\overline{U_{\uparrow\uparrow}U_{\downarrow\downarrow}^*} = \frac{1}{2}$, $\overline{U_{\uparrow\downarrow}U_{\downarrow\uparrow}^*} = -\frac{1}{2}$, thus SO averaging brings a factor of $(\frac{1}{2})^2$ and forces the neutral paths to have parallel spins while the spin of the two partners of charged paths must be antiparallel. As a consequence, one finds the cooperon diagrams cancel in pairs as concluded for the case of the magneto- conductance[24](fig.7) such that no exponential corrections to the conductance occurs due to the cooperon. For the diffuson there are only two combinations possible for incoming spin indexes(all up and all down), such that in this case $\text{Var}_d[P_{spinor}(B, \Delta B, t)] = (\rho_{diff}^{spinor}(\Delta B))t^{2/3} + \ln A(B, \Delta B)$. This nice result explains why there are fluctuations with SO, even if there are not exponential corrections to the magneto-conductance. We find again numerically the predicted $t^{2/3}$ scaling (see fig.6). For large ΔB and t one finds the ratio of the variances approaches $\approx \rho_{diff}(\Delta B)/\rho_{diff}^{spinor}(\Delta B) = 2$ (i.e., $1/(2 \text{ times } (1/2)^2)$), in agreement with the numerical results. Now from eqn. 7 one obtains :

$$C(B, \Delta B, t) = \frac{1}{2} \{ [(\rho_{coop}(2B + \Delta B) + \rho_{diff}(\Delta B)) - (\rho_{mc}(B) + \rho_{mc}(B + \Delta B))] t^{2/3} + S(B, \Delta B) \} \quad (9)$$

where $S(B, \Delta B) = \ln A(B, \Delta B) - \ln A'(B + \Delta B) - \ln A'(\Delta B)$, are logarithmic corrections from the prefactors. $\rho_{mc}(B)$ defines the magneto-conductance, $\overline{[J^*(B)J(B)]^n} = \exp\{\sum \frac{n^i}{i!} C_i [\ln J^*(B)J(B)]\} = A(n, B, \Delta B) \exp(4 \ln n + \rho_{mc}(B)n(n^2 - 1)t)$ [24].

From eqn.9 apart from the predicted $t^{2/3}$ scaling, one can gain qualitative and quantitative understanding of the decaying behavior of the correlation function for small ΔB . With the explicit field dependence of the cooperon and diffuson given by:

$$\rho_{fluctuations} = (2^2/3)(\cos((2B + \Delta B)/B'_c) + \cos(\Delta B/B'_c) + 1)\rho(B = 0)t^{2/3} \quad (10)$$

and

$$\rho_{mc} = (1/3)[(2 + \cos(B/B_c)) + (1/3)(2 + \cos((B + \Delta B)/B_c))]\rho(B = 0)t^{2/3} \quad (11)$$

one obtains:

$$C(B, \Delta B, t) = \frac{1}{6} [8 \cos((B + \Delta B)/B'_c) \cos(B/B'_c) - 2 \cos((2B + \Delta B)/2B_c) \cos(\Delta B/2B_c)] \text{Var}_d[F(0, t)] \quad (12)$$

Here, we have ignored the logarithmic corrections from eqn.9. For $B = 0$ and $\Delta B = 0$ one has $C(B = 0, \Delta B = 0, t) = \text{Var}_d[F(0, t)]$, with $\text{Var}_d[F(0, t)] = \rho(B = 0)t^{2/3}$, in accordance with eqn.7. Now, for small ΔB and $B \approx 0$, one can assume that $B'_c \cong B_c$ [24]:

$$C(B \approx 0, \Delta B, t) = [1 - \frac{1}{2}(\Delta B/B_c)^2] \text{Var}_d[F(0, t)] t^{2/3} \quad (13)$$

This behavior is qualitatively seen even for $B \neq 0$ in fig.3, where for given B one observes a faster decay with increasing t (recall that $B_c = \pi/t^{3/2}[\phi_o]$). This is even more clearly to see with SO. In that case one has:

$$C_{spinor}(B, \Delta B, t) = \frac{1}{2} [(\rho_{diff}^{spinor}(\Delta B) - 2\rho_{mc}^{spinor}) t^{2/3}] \quad (14)$$

$$C_{spinor}(B, \Delta B, t) = \frac{1}{2} [(2^2/2)(\cos(\Delta B/B'_c) + 1)] - 2\rho_{mc}^{spinor} t^{2/3} \quad (15)$$

again an expansion in small ΔB :

$$C_{spinor}(B, \Delta B, t) = (1 - \frac{1}{2}(\Delta B/B_c)^2) \rho_{mc}^{spinor} t^{2/3} \quad (16)$$

On the other hand, without SO, numerical calculations (fig.3), shows a very slow decaying, such that for the field range less than ϕ_o/ℓ^2 , there are not enough ensembles to form, such that even if the field correlation function decays, and a decorrelation field can be defined, it is not possible to define a large enough number of samples such that averaging over the field, could be equivalent to averaging over disorder (samples), a matter that has caused some confusion. With SO the decaying of the correlation function is dominated by an even much bigger field scale unknown to us.

Although the replica results are valid quantitatively for $1+1$ dimensions, nevertheless, $2+1$ dimensions also has a bound state, although weaker, with a smaller positive MC. Therefore, our results apply qualitatively to three dimensional hops[32].

6 Discussion and Conclusions

Summarizing the results of this work, we have found that the log-conductance in the DIR of mesoscopic samples is non-ergodic in the mean square sense and the fluctuations of the log-conductance are non-ergodic in the sense that sample to sample fluctuations are larger than magnetic field fluctuations. Without SO due to the ever increasing magneto-conductance with the field[24, 31], only quasi stationarity can be achieved for fields larger than a certain hopp dependent field $B_s(t)$, whereas with SO due to the small magneto-conductance and its rapid saturation[24, 32], $B_s(t)$ is negligible, so that the process in that case can be regarded as stationary independent of t . The field $B_s(t)$ is obtained numerically by varying B_i until one observes that $\sigma_{mss}(B_f) \approx \text{Var}_d(\langle F(\mathcal{H}, B) \rangle)$. This different behavior with and without SO turns out to have a remarkable influence on the behavior of the two variances that define the sample to sample or disorder fluctuations and the field fluctuations, as their values depend on B_i . It is only when the averaging interval is taken as $\Delta B = [B_s(t), B_f]$, i.e., when quasi-stationarity is achieved, that the proper comparison of the variance can be made. In experiments the hopp values were small, and therefore $B_s(t)$, was small. However, in order to compare the magnitude of the fluctuations defined by eqn.5 and eqn.6 with experiments one must be aware that their values are sensitive to $[B_s(t), B_f]$, an important point indicated by our results. The experiments of [5] were done without SO; therefore, in order to make a rigorous comparison with our results we have to know the experimentally taken interval $[B_s(t), B_f]$ for the evaluation of this quantities. For the small values of t , according to our results, the influence of B_i is small and one should expect $B_s(t) \approx 0$ and indeed we find the ratio of eqn.5 and eqn.6 predicts a value similar to their experiments. On the other hand, the scaling of the disorder fluctuations with the hopp length t , as shown in fig.4, agrees with the experimental values obtained by Orlov et al[33](see fig.45 in the review of Kramer and Mackinnon). It is interesting to observe that both the variance with disorder and the one with field appears to have the same functional form. A more careful comparison can be done by taking into account all predictions of this work. In the case with SO, experimental studies of fluctuations are lacking. To our knowledge the only work where field fluctuations in the strongly insulating regime have been observed in samples with SO is the work of Hernandez and Sanquer [22]. Here our predictions give a clear qualitative way of differentiating the cases with and without SO and highlight a way of evaluation of

experimental data.

From the theoretical point of view, we have derived two important objects: the *cooperon* and the *diffuson* which are the weak localization analogs in the DIR. They allow us to explain some features of the correlation function like the qualitative behavior for small ΔB , and numerically the weak decaying behavior on the scale (ϕ_o/ℓ^2) , that according to the Slutski's theorem, is responsible for the non-ergodic behavior as we have found. In order to experimentally observe persistent diffuson fluctuations, one has to explore a range of parameters so that there is a saturation in the average behavior while the wave function shrinkage[13] is still a negligible effect. This range can be defined by the condition $B_c < \hbar/(ea_B N^{1/3}) = B_{orb}$, where a_B is the Bohr radius. B_{orb} is the scale for the orbital shrinkage to be important[4], i.e., when the cyclotron radius becomes of the order of the mean free path ℓ . These conditions have been met in ref. [5] and [4]. Furthermore, according to references[4, 5, 3] the magnetic field cannot induce geometric fluctuations due to changes in the identity of the hop[30]. This finding holds in both two and three dimensions, and therefore we expect the insulating cooperon and diffuson fluctuations should be seen experimentally also in three dimensions. In experiments, one should be aware about the condition of mesoscopic sample, discussed in section 3. Otherwise, trivial self-averaging of spatially different oriented hops can wash out the possibility to observe fluctuations.

There will be a non-ergodic to ergodic behavior in the case when one relaxes the condition of DIR with $\ell \ll \xi < t$ such that there are many impurities within ξ . In this case there will be a diffusing behavior within the length scale ξ so that two overlapping random processes are at work, one ergodic for the diffusing scale ξ and the other non-ergodic for the larger scale t . This question is under study [12]. Finally, we want to stress that the fundamental point of considering correlations in the random processes $F(\mathcal{H}, t)$ due to the crossing of paths can not be relaxed. This approach permitted us to use the full power of the replica theory, which again helped to semi-quantitative and qualitative predictions. Theories like the independent path approximation, where such crossing is neglected, miss the very crucial objects defined by the cooperon and the diffuson.

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"Fluctuations of $\ln J(B,t)$ "

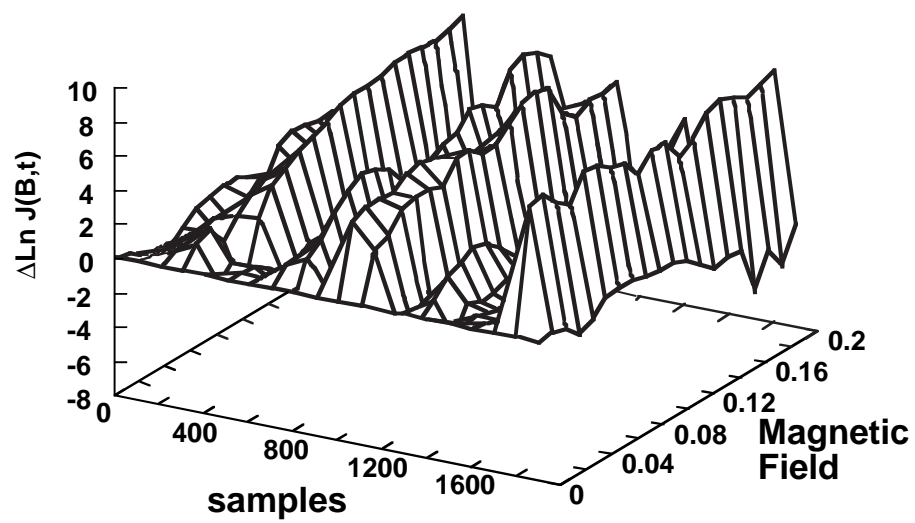


Figure 1: Field Conductance fluctuations of $\ln|J(B,t)|$ and $\ln|I(B,t)|$ for different realizations of disorder or sample number and $t = 15$

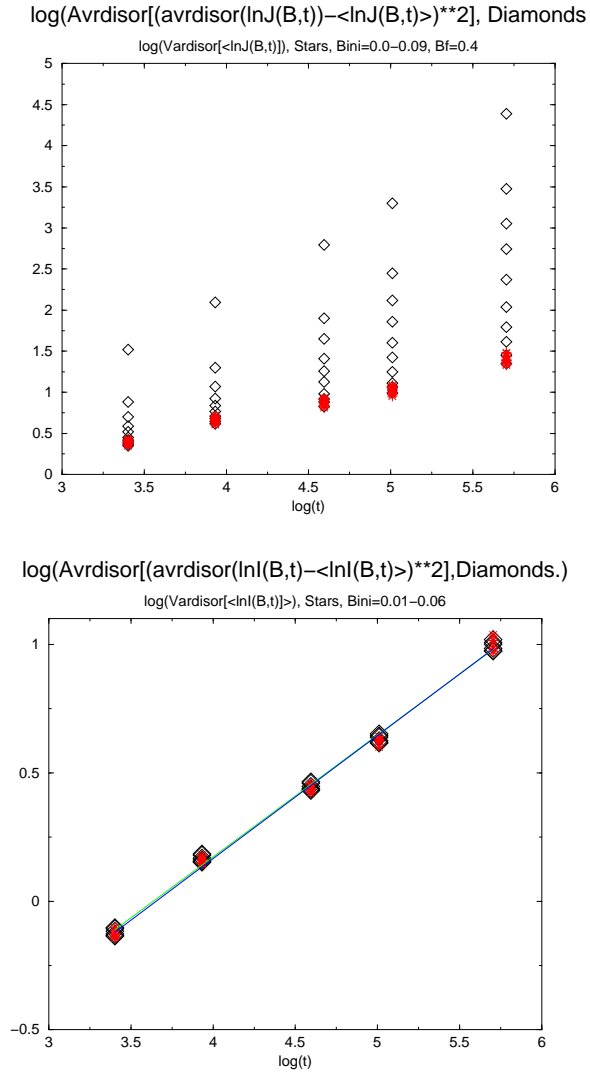


Figure 2: Quasi Stationary behavior without SO. Determining $B_s(t)$ for $t=30,50,100,150,300$. Diamonds converge to stars symbols as the field B_i grows from zero to 0.09 in 0.01 steps. In the case with SO diamonds and stars fall together independent of B_i and t indicating stationarity with $B_s \approx 0$

Figure 3: Correlations functions $C(B, \Delta B, t)$ and $C_{spinor}(B, \Delta B, t)$ as a function of $\Delta B(\phi/\phi_o)$ for a given value of B from $B = 0.0$ to $B = 0.09$. For each value of B , there are three curves corresponding to three values of $t(30, 100, 300)$. One sees a tendency to faster decaying with increasing t .

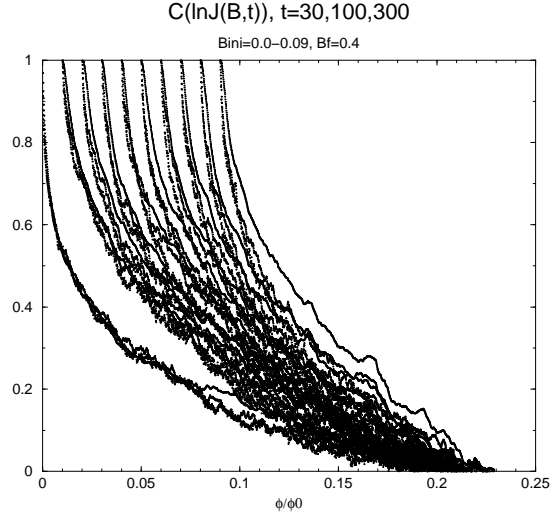


Figure 4: The figure depicts the variance as defined by Eq.5 and Eq.6 as a function of t . The effect of changing the values of B_i from 0 to 0.08 is clearly seen until saturation is attained. Notice that there is a crossing due to a stronger with t increasing dependence on $B_i(t)$ on eqn.5 than in eqn.6. In the case with SO, both quantities have a weak dependence on B_i and the crossing effect is absent.

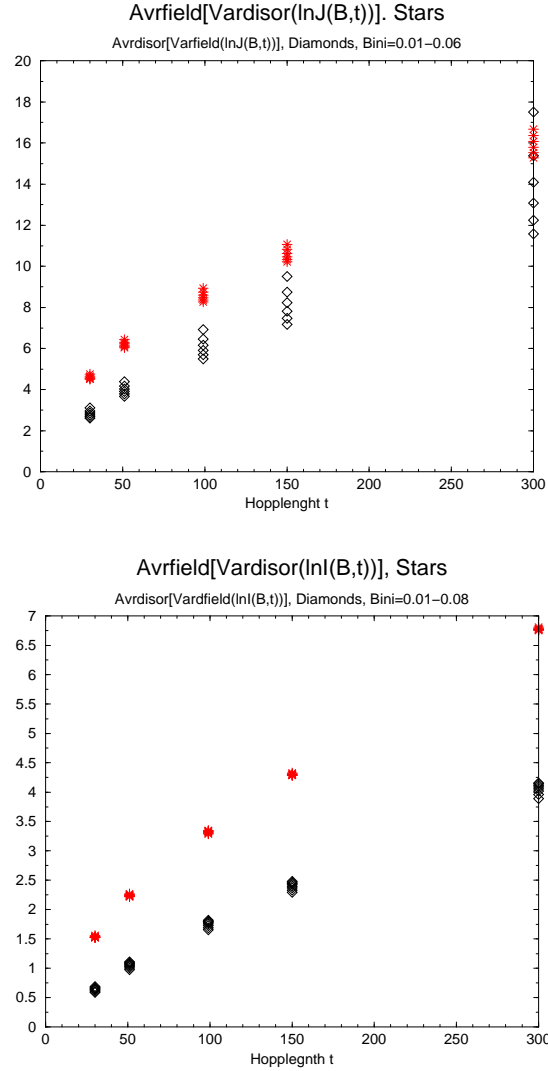


Figure 5: The variance in disorder of $P(B, dB, t)$ and $P_{spinor}(B, dB, t)$ as a function of $dB(\phi/\phi_o)$ for $t = 30, 100, 300$ and $B = B_i = 0.0 - 0.09$.

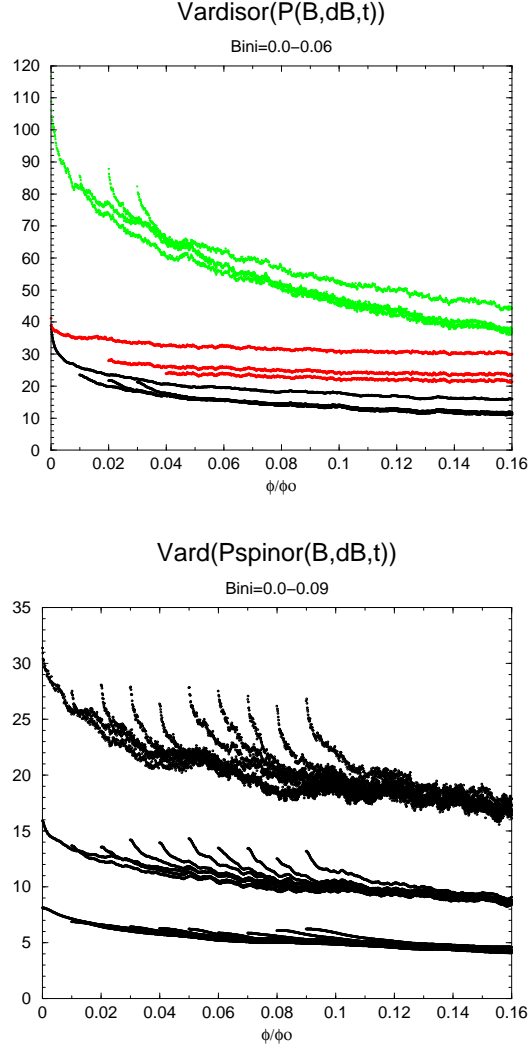


Figure 6: The figure shows the expected $t^{2/3}$ dependence from eqn.8. Each line corresponds to a different ΔB . The upper line corresponds to $\Delta B = 0$. With increasing ΔB the lines move downward and saturate for big enough ΔB . One observes small modulations which are interpreted as deriving from the normalization factors in the Bethe Ansatz.

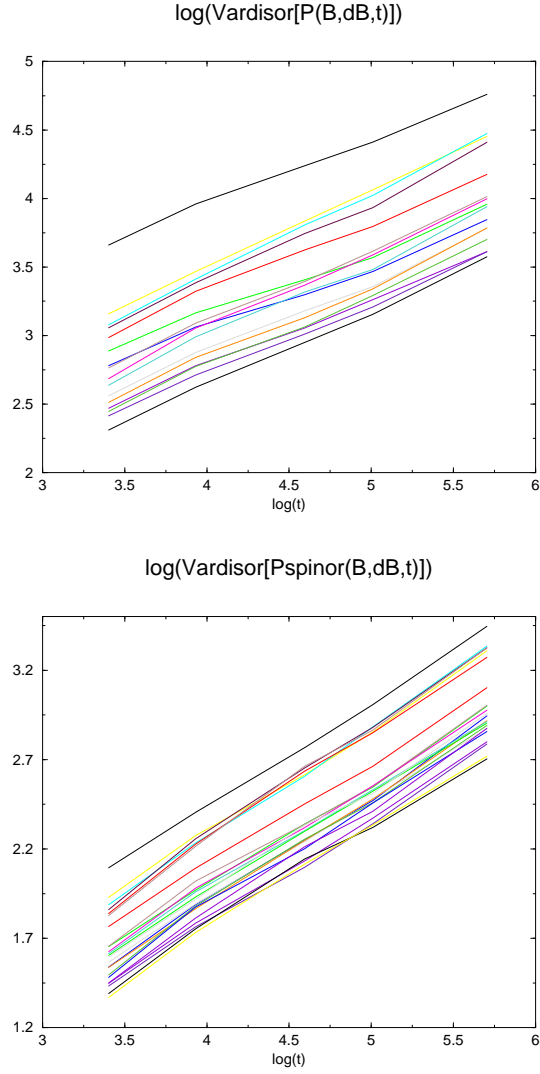


Figure 7: The Cooperon and Diffuson without SO

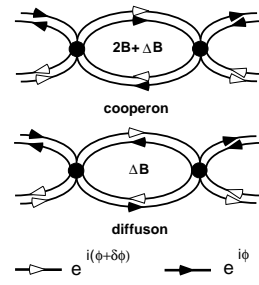
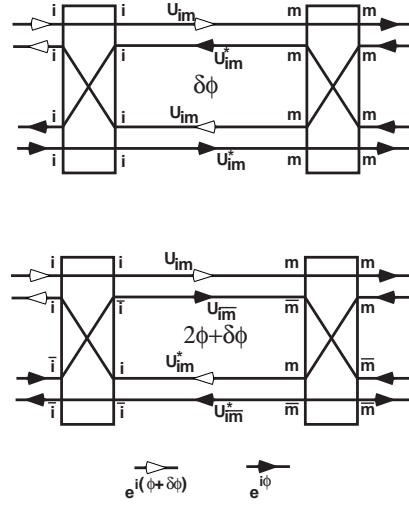
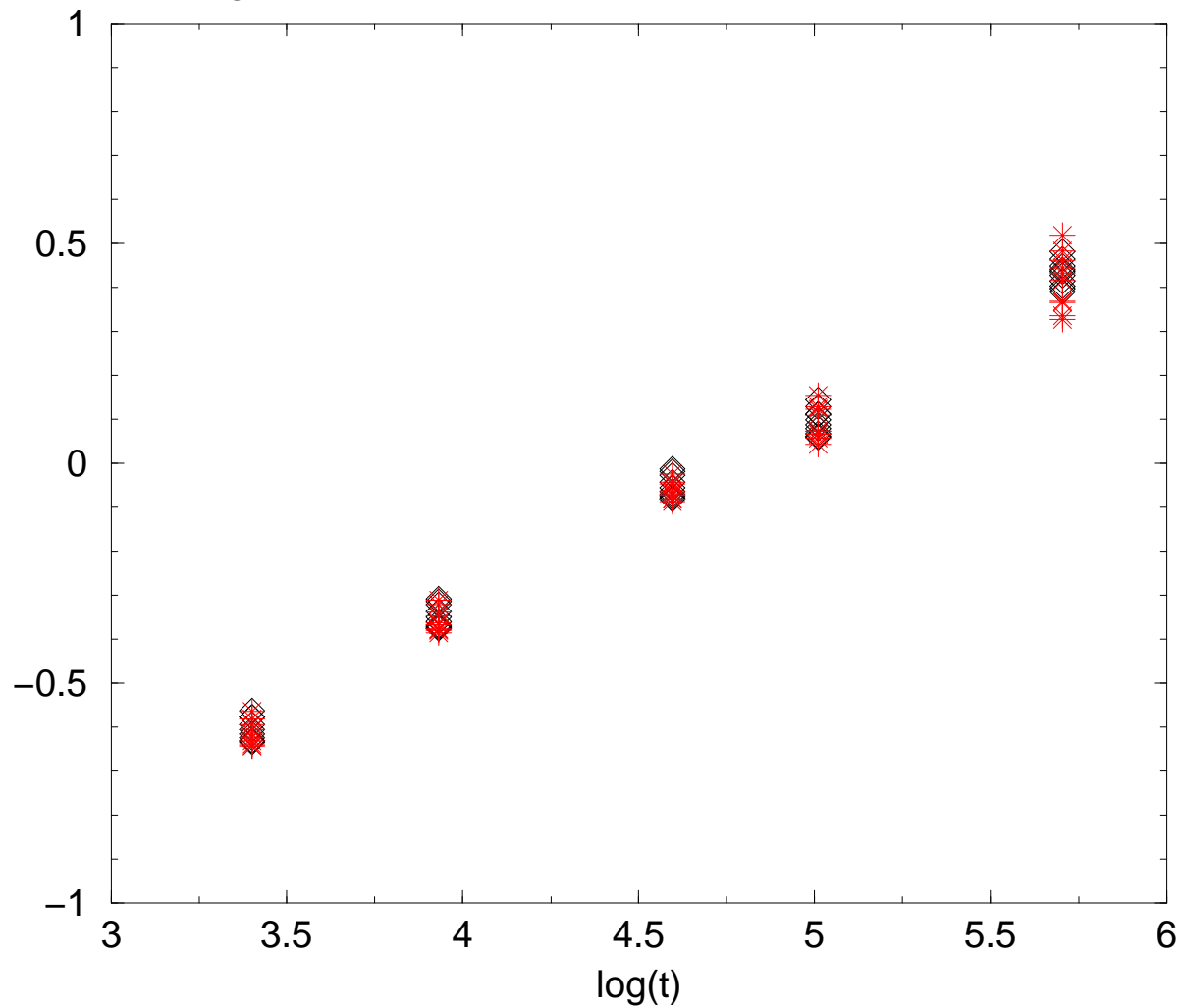


Figure 8: The Diffuson and the Cooperon with SO



$\log(\text{Avrdisor}[(\text{avrdisor}(\ln I(B,t)) - \langle \ln I(B,t) \rangle)^2])$, Diamonds

$\log(\text{Avrdisor}[\langle \ln I(B,t) \rangle])$, Stars, Bini=0.0–0.09, Bf=0.4



Vardisor[$\ln J(B, +dB, t) + \ln J(B, t)$], Stars

Vardisor[$\ln J(B, +dB, t)$] + Vardisor[$\ln J(B, t)$], Squares

